PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

INDEPENDENT PUBLIC SCHOOL

MATHEMATICS SPECIALIST UNITS 1&2
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

Let S be the set of integers between 1 and 79 inclusive.

(a) Determine how many of the integers in S are

    (i) a multiple of 4. (1 mark)

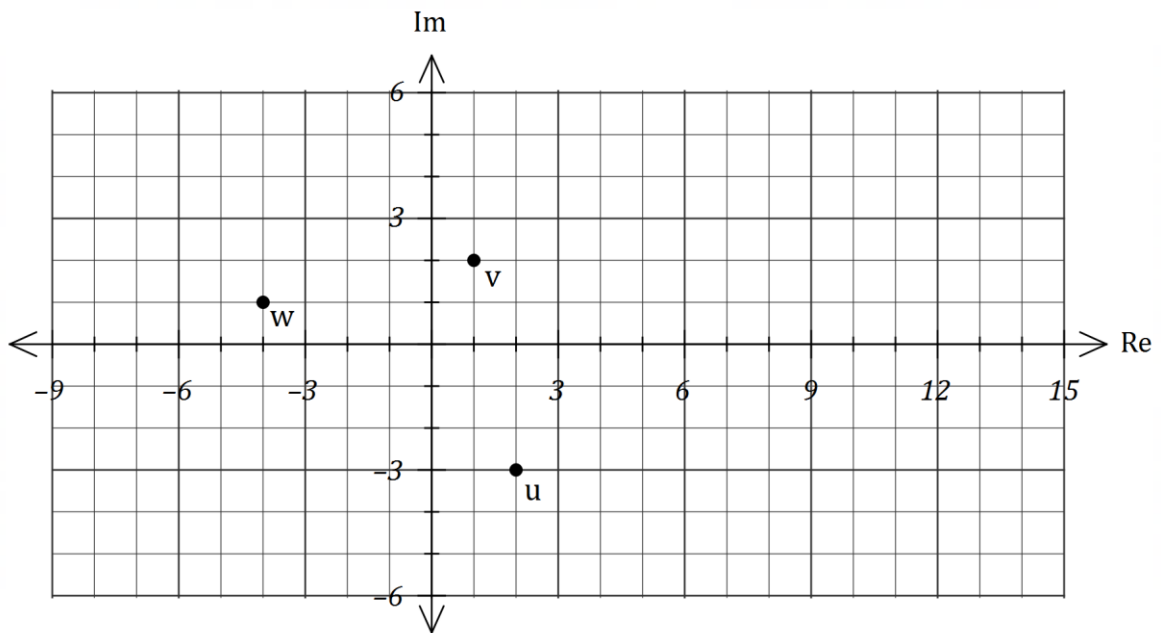   (ii) a multiple of 4 or a multiple of 3. (3 marks)

(b) Integers are selected one at a time, at random and without replacement from S. After how many selections can you be certain that the squares of at least three of the integers selected will share the same last digit? Justify your answer. (3 marks)

Question 10 (7 marks)

The location of u, v and w in the complex plane are shown below.

(a) Plot and label the following on the same diagram:

(i) $z_1 = u - 2w$. (1 mark)

(ii) $z_2 = \bar{u} - 2\bar{w}$. (1 mark)

(iii) $z_3 = v - 4 + 2i$. (1 mark)

(iv) $z_4 = wi$. (1 mark)

(b) The complex number v is a solution to the equation $z^2 + az + b = 0$. Determine the value of the real constant a and the real constant b. (3 marks)

Question 11 **(8 marks)**

Quadrilateral $PQRS$ has vertices $P(4, 1), Q(10, 5), R(9, -4)$ and $S(2, -3)$.

(a) Show that $\overrightarrow{PR}$ is perpendicular to $\overrightarrow{QS}$. (3 marks)

(b) Show that $|\overrightarrow{QS}| < |\overrightarrow{QR}| + |\overrightarrow{RS}|$. (3 marks)

(c) Determine the angle between $\overrightarrow{QR}$ and $\overrightarrow{RS}$ to the nearest degree. (2 marks)

Question 12 **(7 marks)**

Triangle ABC is transformed by matrix $T = \begin{bmatrix} 3 & 0 \\ 0 & -4 \end{bmatrix}$ to $A'B'C'$ and then rotated 90° clockwise about the origin to $A''B''C''$.

The coordinates of A and B are $(-3, 5)$ and $(4, 7)$ respectively and the area of ABC is 35.5 square units.

(a) Determine the coordinates of A'. (1 mark)

(b) Determine matrix S that represents a 90° clockwise rotation about the origin. (1 mark)

(c) Determine the coordinates of B''. (2 marks)

(d) The coordinates of C'' are $(16, -3)$. Determine

   (i) the coordinates of C. (2 marks)

   (ii) the area of triangle $A''B''C''$. (1 mark)

Question 13 **(7 marks)**

The air pressure in a tank can be modelled by the equation

$$p = a + b\cos\big(c(t+d)\big) \text{ for } 0 \le t \le 24$$

where p is the pressure in kPa, t is the time in hours after midnight and all other variables are positive constants.

The air pressure first reached a minimum of 92 kPa when $t = 0.5$ h and then rose during the next 3 hours to a maximum of 116 kPa before decreasing again.

(a)   Determine the value of each of the positive constants a, b, c and d.   (4 marks)

(b)   Use the model to determine
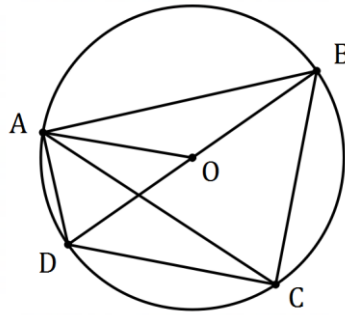
(i)   the air pressure at 6 pm.   (1 mark)

(ii)   the time of day, to the nearest minute, that the pressure first reached 111 kPa.   (2 marks)

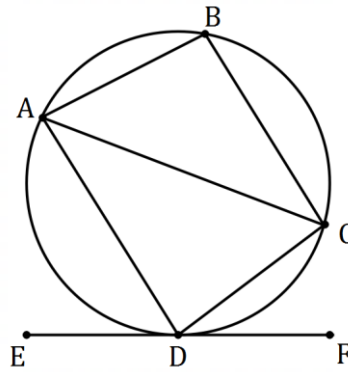**Question 14**   **(9 marks)**

(a)   In the diagram below, A, B C and D lie on the circle with centre O. If $\angle BDC = 38°$ and
        $\angle ACD = 15°$, determine with reasoning $\angle DAC$ and $\angle AOB$.   (4 marks)

(b)   In the diagram below, $ABCD$ is a cyclic quadrilateral and EF is a tangent to the circle at D. If $\angle BAC = 52°$, $\angle ADE = 64°$ and $\angle ADC = 84°$ prove that AD is parallel to BC.   (5 marks)

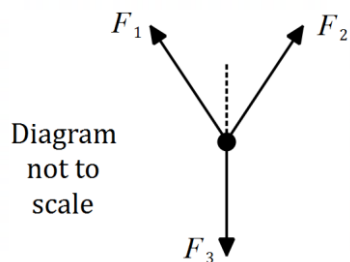**Question 15** **(7 marks)**

(a) Two forces of magnitudes 120 N and 170 N are inclined at 25°. Determine the magnitude of their resultant. (3 marks)

(b) A weight is in equilibrium, suspended by two ropes. The diagram below shows forces F_1 and F_2 acting in the ropes and F_3 exerted by the weight. If F_1 has a magnitude of 20 N and acts upwards at an angle of 55° to the vertical and F_2 acts upwards at an angle of 15° to the vertical, determine the magnitude of F_2 and F_3. (4 marks)

Diagram not to scale

Question 16 **(7 marks)**

A system of equations is given by

$$4x + ay - 9 = 0$$
$$-2x + 3y + 3 = 0$$

(a) Let the constant $a = -5$.

    (i) Express the system in matrix form $AX = B$, where X and B are column matrices.

        (2 marks)

    (ii) Determine A^{-1} and demonstrate use of matrix algebra to solve the system for X.

        (3 marks)

(b) Determine the value of a for which the system has no solution and comment on the relationship between the two lines that form the system when a has this value. (2 marks)

Question 17 **(7 marks)**

(a) Show that the sum of the recurring decimals $0.\overline{16} + 0.\overline{351}$ is a rational number. (3 marks)

(b) Use algebraic reasoning to prove that if m is one more than a multiple of 3, then $m^2 + 2$ will always be a multiple of 3. (4 marks)

See next page

Question 18 **(6 marks)**

The points A, B, C, D and E lie in that order on a circle marked out on a level school playing field. D is due east of E and C is due north of D. The bearing of A from C is 242° and the bearing of B from A is 044°.

(a) Sketch a diagram to show the approximate positions of A, B, C, D and E. (1 mark)

(b) Explain why $\angle CAE$ is a right angle. (1 mark)

(c) Determine the bearing of

(i) E from A. (2 marks)

(ii) B from D. (2 marks)

Question 19 **(8 marks)**

A boat that travels at 30 km/h in still water starts from the corner X of an isosceles triangle XYZ where $XY = YZ = 15$ km and $XZ = 23$ km and describes the complete course $XYZX$ in the least possible time. A current of 8 km/h runs in the direction $\overrightarrow{ZX}$.

(a)   Determine the speed of the boat between X and Y. (5 marks)

(b)   Determine the time the boat takes to complete the course to the nearest minute.
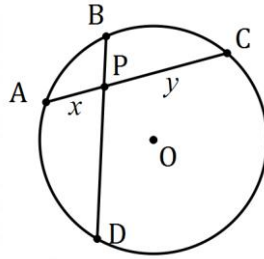
(3 marks)

Question 20 **(9 marks)**

Eight congruent cubes, each of a different colour, are to be arranged in a straight line. One of the cubes is green and another is pink.

(a) Determine how many different arrangements of all 8 cubes are possible if

    (i) there are no restrictions. (1 mark)

    (ii) the green cube must not be next to the pink cube. (3 marks)

(b) Determine how many different arrangements of 3 cubes chosen from the 8 are possible if

    (i) there are no restrictions. (1 mark)

    (ii) the arrangement must include the green cube. (2 marks)

    (iii) the arrangement must not have the green cube next to the pink cube. (2 marks)

Question 21   **(9 marks)**

Points A, B, C and D lie on a circle such that chords AC and BD intersect inside the circle at P as shown. Let $AP = x$ and $CP = y$.

M is the midpoint of AC and the circle has radius r and centre O so that $OP = d$ and $OM = k$.

(a)   Sketch triangle OAC to show this information.   (2 marks)

(b)   Express the length of AM and PM in terms of x and y.   (2 marks)

(c)   Use triangle OAM to write a relationship between k, r, x and y, and use triangle OPM to write a relationship between k, d, x and y. (2 marks)

(d)   Show that $r^2 - d^2 = AP \times PC$. (2 marks)

(e)   If the radius of the circle is 65 cm, $BD = 110$ cm and $BP = 11$ cm, determine the distance of P from the centre of the circle. (1 mark)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____